Leader Notes: I've Seen the Light!

Explore/Explain Cycle III

Purpose:

Solve a problem by collecting and analyzing data leading to the use of a rational function as a model. Create numerical and graphical representations to analyze the data using technology.

Descriptor:

Participants will explore the relationship between the intensity of light and the distance from the light source by using a light sensor and calculator-based laboratory to collect data. They will create tabular and graphical representations using a graphing calculator, spreadsheet, and TI-Interactive. Participants will compare and contrast the use of these technologies and their effectiveness in representing the data and communicating the results of the data analysis.

Duration:

2 hours

TEKS:

a(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.1(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

2A.4(A) Identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).

2A.4(B) Extend parent functions with parameters such as a in $f(x) = \frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.

2A.10(B)	analyze various representations of rational functions with respect to problem situations;
2A.10(C)	determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;
2A.10(D)	determine the solutions of rational equations using graphs, tables, and algebraic methods;
2A.10(E)	determine solutions of rational inequalities using graphs and tables;
2A.10(F)	analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and
2A.10(G)	use functions to model and make predictions in problem situations involving direct and inverse variation.

TAKS Objectives Addressed by these Algebra 2 TEKS:

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 10: Mathematical Processes and Mathematical Tools

Technology:

- CBL2 or other calculator-based data collection device
- Light probe or sensor
- Graphing Calculator
- Spreadsheet
- TI-Interactive
- Graph linking capability, such as TI-Connect or Casio Program-Link

Materials:

Advanced Preparation: Be sure that the flashlight batteries are fresh.
Have extra batteries for the CBL2 on hand.

Presenter Materials: projector for graphing calculator and computer demonstration

Per group: CBL2, light probe, flashlight with fresh batteries, 2 or 3 meter sticks OR metric tape measure, graph link cable appropriate to type of calculator being used

Per participant: graphing calculator, activity sheets

Leader Notes:
If you shine a flashlight at a far wall in a dark room, the light will spread out and diffuse over most of the wall. As you get closer to the wall, the light covers a smaller area but is brighter and

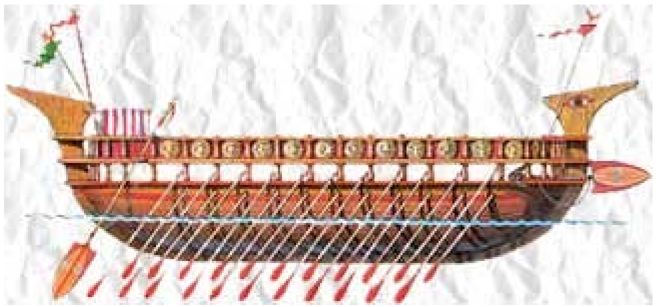
more clearly defined. The brightness of the light is called its intensity and is measured in watts per unit area, usually square meters or square centimeters.

The relationship is an inverse-square variation relationship of the form $y = \frac{k}{x^2}$, which is one form of rational function. Participants will be asked to gather the data and analyze it on their own. During the debrief of the Explain phase, several ways of analyzing the data will be discussed and participants will be asked to identify comparative advantages and disadvantages of each method.

Participants will be using the CBL2 or equivalent calculator-based laboratory data collection device. Participants' instructions for using the light probe are based on the CBL2 and the DataMate Application. DataMate is an APP which can be downloaded directly from the CBL2 to the graphing calculator by linking the two devices, setting the calculator to LINK-RECEIVE, then pressing the "TRANSFER" button on the CBL2.

Explore

Posing the Problem:



A Phoenician boat captain was enjoying the cool Mediterranean breeze as his boat sailed from Tyre to Carthage with another shipment of purple dye for the king. The 1200-mile voyage was not easy. The captain smiled to himself as he thought of the many boats from other nations that became lost at sea attempting to make this journey. He and his Phoenician counterparts had a leg up on the competition- they knew how to use the stars to navigate. The captain looked up at the night sky, noticing the stars to make sure he was on course. He was always amazed by the variety of stars, some blue and white, some yellow and red. Some bright, some faint.

We know today that the brightness of a star depends on several factors. One factor is the distance between the Earth and the star. These distances are difficult to measure, so they must be calculated. In order to calculate these distances, astronomers must first know the relationship between the intensity of starlight and the distance between the Earth and the star.

Obtaining and Analyzing the Data:

To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, use a flashlight to simulate a star and use a light-intensity sensor to measure the intensity of the light for varying distances.

Attach a light sensor to your data collection device and graphing calculator. Run a program, such as the DataMate App, that measures intensity of light to collect data. One person in the group should hold the light sensor as another person walks towards the sensor with the flashlight.

1. **Use the light sensor to collect data in intervals of 0.1 meter. Record your data in the table.**
Participants should determine their own distances. The light sensor (depending on calibration) can detect between 0 and 1 milliwatt per square centimeter. Participants will need to find the first 0.1-meter distance that gives an intensity reading below 1 then collect data from there. The brighter the flashlight, the further away from the flashlight this initial reading will be.

Facilitation Questions

- What part of the light source is the light probe measuring?
The light probe measures the intensity of the light striking it perpendicular to the lens.
- Where should you point the light probe in order to get consistent measurements?
Aim the lens of the light probe toward the center of the flashlight to insure that you are measuring the most direct part of the light beam.

Sample data:

Distance (D) (m)	Intensity (I) (mW/cm^2)
0.6	0.7454
0.7	0.5657
0.8	0.4588
0.9	0.3199
1.0	0.2538
1.1	0.2149
1.2	0.1751
1.3	0.1479
1.4	0.1333
1.5	0.1236
1.6	0.1100
1.7	0.0973
1.8	0.0906
1.9	0.0808
2.0	0.0750

2. **Using an appropriate technology, generate a scatterplot of your data. Sketch your scatterplot.**
Actual scatterplots may vary depending on the technology chosen by the participants.

Facilitation Questions

- What are your independent and dependent variables?
Distance is the independent variable and intensity is the dependent variable.
- What are the domain and range of your data?
Answers may vary. According to the sample data, the domain of the distance is from 0.6 meters to 2.0 meters and the range of the intensity is from 0.0750 to 0.7454 milliwatts per square centimeter.

3. **Find an appropriate function rule to model your data. Test the rule over your scatterplot. Sketch your graph.**
   Function rules will vary depending on the data collected. A function rule modeling the sample data is $y = \frac{0.273}{x^2}$.

> **Facilitation Questions**
> - What type of function does this data set appear to represent?
>   *The data appear to curve like an inverse variation (rational) function.*
> - Are the *y*-values increasing or decreasing as the *x*-values increase?
>   *The y-values are decreasing as x increases.*
> - Is there a constant rate of change?
>   *No.*
> - What other kinds of parent functions are there in this family?
>   *Any function* $y = \frac{k}{x^n}$*, where k and n are constants*
> - How can you determine the values of the parameters for that kind of function?
>   *To find the value of k, multiply* $x^n y$*. If the value of k is close for all x-y ordered pairs, then the curve is likely to be a good fit.*

4. **A plant will be placed 275 centimeters from the light source. What intensity of light will it receive? Justify your answer.**
   Based on the sample data, set up and simplify the equation $y = \frac{0.273}{(2.75)^2} \approx 0.036 \frac{mW}{cm^2}$.

5. **A particular solar cell needs to receive at least 0.4 milliwatts per square centimeter of light to generate enough electricity to power a small toy. How far from the light source should the solar cell be placed in order to begin powering the toy? Justify your answer.**
   Based on the sample data, set up and solve the inequality $0.4 \geq \frac{0.273}{x^2}$. *The solar cell should be placed about 0.826 meters, or 82.6 centimeters from the light source.*

Explain

In this phase, use the debrief questions to prompt participant groups to share their responses to the data analysis. At this stage in the professional development, participants should be familiar with using the graphing calculator, a spreadsheet, and TI-Interactive. If none of the participant groups uses one of these three methods, ask them how they could have used that method to analyze the data. This information is important to the discussion of relative advantages and disadvantages of different types of technology. The reasons that a participant group did not choose a particular technology are as important (if not more so) than the justifications a group gives for the technology that they did choose.

1. **How did you develop your scatterplot? Why did you choose this method?**
   Ask participants to discuss their methods and their reasons for making that choice. If none of the participants chooses one of the three technologies (graphing calculator, spreadsheet, or TI-Interactive), ask participants why no one made that choice.

   See "Technology Tutorial: I've Seen the Light!" for details.

2. **How did you develop your function rule? Why did you choose this method?**
   Ask participants to discuss their methods and their reasons for making that choice. If none of the participants chooses one of the three technologies (graphing calculator, spreadsheet, or TI-Interactive), ask participants why no one made that choice.

   See "Technology Tutorial: I've Seen the Light!" for details.

3. **How did you solve the problems? Why did you choose this method?**
   Ask participants to discuss their methods and their reasons for making that choice. If none of the participants choose one of the three technologies (graphing calculator, spreadsheet, or TI-Interactive), ask participants why no one made that choice.

   See "Technology Tutorial: I've Seen the Light!" for details.

Note to Leader: *Record or have a participant volunteer record the responses to Questions 4, 5, and 6 on chart paper to use in the Elaborate phase of the professional development.*

4. **What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?**
   Responses may vary.

   The data analysis can be done in a few keystrokes. The power to set your own parameters and graph the function rule empowers the participant to use numerical analysis to calculate meaningful parameters such as a constant of variation. The graphical analysis features of the calculator make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.

   However, the small screen is difficult to see, and the axes in the window cannot be labeled.

5. **What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?**
   Responses may vary.

   The regression equation is calculated quickly on the spreadsheet. The axes can be clearly labeled with numbers and text labels. Labeled axes help the participant to use the graph to estimate solutions to problems that can be solved graphically. The graph can be enlarged or *reduced then copied and pasted into other computer documents such as a Word or PowerPoint document to communicate the solution to a problem.*

However, the participant is limited to the regression equations available in the spreadsheet. There are no graphical analysis features in most spreadsheets, so only estimates rather than exact solutions can be obtained graphically.

6. **What are the relative advantages and disadvantages of using TI-Interactive to solve this problem?**
   Responses may vary.

   Like the graphing calculator, data analysis can be done with a few keystrokes and clicks. The function editor enables participants to set their own rational function, empowering them to choose parameters that make physical sense in the context of the problem. The graphical analysis features of TI-Interactive make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.

   Like the spreadsheet, axes can be labeled numerically and with text. The graphs are cleaner and can be copied and pasted into other computer documents.

7. **What TEKS does this activity address?**
   Participants should brainstorm a list of TEKS that they believe they have covered in this activity. The Leader Notes contain a comprehensive list of the TEKS addressed in this phase of the professional development. If participants do not mention some of these TEKS, then ask them how the activity also covers them.

8. **How does the technology that you used enhance the teaching of those TEKS?**
   Responses may vary. However, participants should note that using technology enables them to explore a mathematical concept to a much deeper level. For example, in this activity, using a spreadsheet or the List Editor in a graphing calculator or TI-Interactive allows participants to make quick computations that allow them to determine inverse variation (xy is a constant value) or inverse square variation relationships (x^2y is a constant value).

   Technology makes rich mathematics accessible to a variety of learning styles. For example, students can use a graphing calculator to solve equations and inequalities via tables and graphs rather than merely relying on traditional symbolic manipulation.

I've Seen the Light!: Intentional Use of Data

1. At the close of *I've Seen the Light!*, distribute the **Intentional Use of Data** activity sheet to each participant.

2. Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Prompt the participants to also identify two key questions that were emphasized during this activity. Allow four minutes for discussion.

Facilitation Questions

- Which mathematics TEKS formed the primary focus of this activity?
- What additional math TEKS supported the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

3. As a whole group, share responses for two to three minutes.

4. As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this activity. Allow three minutes for discussion.

Facilitation Question

- What attributes of the activity support the level of rigor that you identified?

5. As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.

6. Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will be used during the elaborate phase as prompts for generating attributes of judicious users of technology.

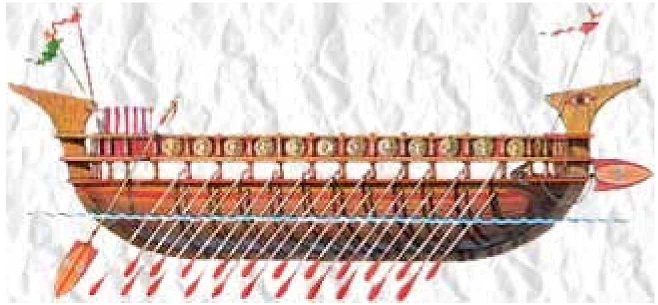
Facilitation Questions

- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per participant?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per small group of participants?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) for the entire group of participants?
- How would this activity change if we had used graphing calculators instead of computer-based applications?
- How would this activity change if we had used computer-based applications instead of graphing calculators?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

Sample Responses:

TEKS		*a(5), a(6), 2A.1B, 2A.10B, 2A.10C, 2A.10D, 2A.10E, 2A.10F, 2A.10G*	
Question(s) to Pose to Students	**Math**	*How did you know what type of function could model the data?* *How did you generate your function rule?*	
	Tech	*How did the technology enable you to collect data?*	
Cognitive Rigor		Knowledge	√
		Understanding	√
		Application	√
		Analysis	√
		Evaluation	√
		Creation	√
Data Source(s)		Real-Time	*The CBL2 generates real-time data*
		Archival	
		Categorical	
		Numerical	
Setting		Computer Lab or CBL2 for each student	*Each student uses the computer or CBL2 to generate their own data.*
		Mini-Lab or CBL2 for each group	*In groups students take turns or groups switch out.*
		One Computer or CBL2 for entire class	*A student operates the control as other students read directions, entire class records data.*
		Graphing Calculator	*Could be used to enter data and find relationships.*
		Measurement-Based Data Collection	*Could be done at stations or individually.*
Bridge to the Classroom		*This activity transfers directly to the classroom with the only modifications being the settings addressed above.*	

I've Seen the Light!



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I've Seen the Light!: Intentional Use of Data

TEKS			
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	Tech		
Cognitive Rigor		Knowledge	
		Understanding	
		Application	
		Analysis	
		Evaluation	
		Creation	
Data Source(s)		Real-Time	
		Archival	
		Categorical	
		Numerical	
Setting		Computer Lab	
		Mini-Lab	
		One Computer	
		Graphing Calculator	
		Measurement-Based Data Collection	
Bridge to the Classroom			